Modeling Changes in Brain Pressures, Volumes, and Cerebral Capillary Fluid Exchange: Hydrocephalus

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Outline

- Objective
- Experimental Background and Conceptual Overview
- Brain Pressures: Theory and Results
- Brain Volumes: Theory and Results
- Cerebral Capillary Fluid Exchange
- Future Work and Conclusions
Objective

- Determine mechanism of brain parenchyma compression during hydrocephalus
  - Winston: Extracellular Fluid Drainage
  - Our hypothesis: Extracellular Fluid Absorption
- Develop Compartmental Model of Brain Pressures and Volumes
Experimental Background

Conceptual Compartmental Model

Cranium

Arterial

Brain Parenchyma

CSF + Ventricular

Venous
Cerebral Fluid System

Cerebral Blood Flow

Pa → Pc → Pv → Pvs

Cai

Cv

Rf

R0

CSF + Ventricular System

Brain Parenchyma

Ctiss

Pic

Circuit Analogy

\[ P_a = 17.5 \sin(2\pi t - \frac{\pi}{2}) - 12.5 \sin(4\pi t) + 16000 \]

Brain Pressures: 1\textsuperscript{st} Simulation

Brain Pressures: 2nd Simulation

Brain Pressures: Normal v. Hydrocephalus

- $Amp = 4$
- $Amp = 5.5$
- $R_0 = 7 \Omega$

- Capillary Pressure
- Venous Pressure
- Intracranial Pressure
- Sagittal Sinus Pressure

$R_0 = 7 \Omega$
Brain Pressures: 3\textsuperscript{rd} Simulation

\[ R_0 = 5\Omega \]
Volume Theory

Monro-Kellie Doctrine

\[
\frac{dV_a}{dt} + \frac{dV_v}{dt} + \frac{dV_{tiss}}{dt} + \frac{dV_{CSF}}{dt} = 0
\]

Volume and Compliance Relationships

\[
\frac{dV_a}{dt} = C_{ai} \frac{d}{dt} \left( P_a - P_{ic} \right)
\]

\[
C_{ai} = \frac{1}{K_a \left( P_a - P_{ic} \right)}
\]

\[
\frac{dV_v}{dt} = C_{vi} \frac{d}{dt} \left( P_v - P_{ic} \right)
\]

\[
C_{vi} = \frac{1}{K_v \left( P_v - P_{ic} - P_{v1} \right)}
\]

\[
\frac{dV_{tiss}}{dt} = -C_{tiss} \frac{dP_{ic}}{dt}
\]

\[
C_{tiss} = \frac{1}{K_E \left[ P_{ic} + \left( \frac{P_{ic}}{P_{01}} \right)^2 \right]}
\]

\[
\frac{dV_{CSF}}{dt} = \frac{P_c - P_{ic}}{R_f} - \frac{P_{ic} - P_{vs}}{R_0}
\]

Arterial Volume

\[
\frac{dV_a}{dt} = \frac{1}{K_a} \frac{d}{dt} \left( P_a - P_{ic} \right) \quad \Rightarrow \quad dV_a = \frac{1}{K_a} \frac{d \left( P_a - P_{ic} \right)}{\left( P_a - P_{ic} \right)}
\]

\[
V_a = \frac{1}{K_a} \ln \left( P_a - P_{ic} \right) + V_{ai}
\]
Venous, Parenchyma Volumes

After a similar derivation...

\[ V_v = \frac{1}{K_v} \ln \left( P_v - P_{ic} - P_{v1} \right) + V_{vi} \]

\[ V_{tiss} = -\frac{P_{01}^2}{K_E} \frac{\ln \left( P_{ic}^2 + P_{01}^2 P_{ic} \right)}{2P_{ic} + P_{01}^2} + V_{tissi} \]
Ventricular System Volume

Utilizing the Monro-Kellie Doctrine:

\[
\frac{dV_{CSF}}{dt} = -\frac{dV_a}{dt} - \frac{dV_v}{dt} - \frac{dV_{tiss}}{dt}
\]

\[
dV_{CSF} = -dV_a - dV_v - dV_{tiss}
\]

\[
V_{CSF} = -(V_a - V_{ai}) - (V_v - V_{vi}) - (V_{tiss} - V_{tissi}) + V_{CSFi}
\]
Brain Volumes: 1st Simulation

$R_0 = 10\Omega$
Volume Changes: 1	extsuperscript{st} Simulation

Normal

- Ventricular: 20%
- Arterial: 10%
- Venous: 23%
- Parenchyma: 47%

Hydrocephalus

- Ventricular: 32%
- Arterial: 10%
- Venous: 15%
- Parenchyma: 43%
Brain Volumes: 2nd Simulation

\[ R_0 = 7 \Omega \]

Diagram showing the volume changes over time with respective labels and scales.
Volume Changes: 2\textsuperscript{nd} Simulation

Normal

- Ventricular: 20%
- Arterial: 10%
- Venous: 23%
- Parenchyma: 47%

Hydrocephalus

- Ventricular: 27%
- Arterial: 10%
- Venous: 19%
- Parenchyma: 44%
Brain Volumes: 3rd Simulation

R_0 = 5 \Omega
Volume Changes: 3rd Simulation

Normal

- Ventricular: 20%
- Arterial: 10%
- Venous: 23%
- Parenchyma: 47%

Hydrocephalus

- Ventricular: 25%
- Arterial: 10%
- Venous: 21%
- Parenchyma: 44%
Cerebral Capillary Fluid Exchange

\[ J = \frac{L_p S}{\pi} \left( (P_c - P_{ic}) - (\Pi_c - \Pi_{ic}) \right) \]

Cerebral Capillary: 2\textsuperscript{nd} Simulation

$R_0 = 7\, \Omega$

Capillary v. ICP

Volume Flux across Brain Capillary: Normal v. Hydrocephalus

$R_0 = 7\, \Omega$
Cerebral Capillary: 3rd Simulation

Capillary vs ICP

Pressure (Pa)

Capillary Pressure
Intracranial Pressure

Volume Flux across Brain Capillary: Normal vs Hydrocephalus

Volumetric Flow Rate (m³/s)

R₀ = 5Ω
Conceptual View of Cerebral Capillaries

Under Normal Conditions...

\[ P_c - P_{ic} > \Delta \Pi \]
When the Sagittal Sinus Fails…

\[ P_c - P_{ic} < \Delta \Pi \]

Brain Tissue

With Hydrocephalus…

Cerebral Capillary

\( P_a \)  \( P_{ic} \)  \( P_{c} \)  \( P_v \)

\( H_2O \)

“Extracellular Fluid”

Blood-Brain Barrier
Future Work

- Decipher circuit and determine pressure (voltage) equations

- Collaboration with simulations done in FLUENT and Gambit
Attempt at Solving Pressure (Voltage) Equations

\[ P_a = 17.5 \sin(2\pi t - \frac{\pi}{2}) - 12.5 \sin(4\pi t) + 16000 \]

\[ P_c = \frac{1}{\delta_c} \left[ \delta_f \left( P_v - P_{ic} + P_{vs} \right) + \delta_v P_v \right] \]

\[ \frac{dP_v}{dt} = \frac{1}{K_a \left( P_a - P_{ic} \right)} \left[ \delta_a P_a - \delta_v P_v - \delta_f \left( P_v - P_{ic} + P_{vs} \right) \right] + \left( 1 + \frac{\delta_f}{\delta_v} \right) \left[ \frac{1}{K_v \left( P_v - P_{ic} - P_{v1} \right)} \left( \delta_{vs} P_{vs} - \delta_v P_v \right) \right] \]

\[ \frac{dP_{vs}}{dt} = \frac{1}{K_E \left[ P_{ic} + \left( \frac{P_{ic}}{P_{01}} \right)^2 \right]} \left( \delta_a P_a - \delta_0 P_{ic} - \delta_{vs} P_{vs} \right) - \frac{1}{K_v \left( P_v - P_{ic} - P_{v1} \right)} \left( \delta_{vs} P_{vs} - \delta_v P_v \right) \]

\[ \frac{dP_{ic}}{dt} = \frac{1}{K_E \left[ P_{ic} + \left( \frac{P_{ic}}{P_{01}} \right)^2 \right]} \left( \delta_a P_a - \delta_0 P_{ic} - \delta_{vs} P_{vs} \right) \]

Note: \( \delta = 1/R \)

Conclusion

- Developed Compartmental Model dependent on circuit simulation

- Able to describe mechanism of brain tissue compression

- Winston: Extracellular Fluid Drainage

- Our Hypothesis: Extracellular Fluid Absorption
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